STOCHASTIC MODEL OF THE FLOW MECHANICALLY STIRRED SOLID–LIQUID SYSTEM

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An attempt is made to describe by use of the onedimensional stochastic model the flow system solid phase–liquid stirred by the mechanical rotary mixer. At the assumption that the solid phase is dimensionally homogeneous a diffusion equation has been obtained which is solved under the boundary conditions, characterizing the method of withdrawal of the solid phase from the system. It is demonstrated that the model can express also the relations usually used for description of similar situations (ideal mixing, separation coefficients in stationary state *etc.*) like individual cases.

A number of operations exist in the chemical industry in which a contact of a suspension formed by solid and liquid phases takes place. Typical examples of these operations are crystallisation, dissolving, leaching or suspension of catalysts in heterogeneous chemical reactions.

The most usual device for contact of solid and liquid phases is a vessel with a mechanical mixer. The present knowledge concerning the two-phase system formed by the solid phase and liquid, similarly as that on suspension and homogenisation of solid particles in the batch operated vessels, is relatively extensive. Its survey is given *e.g.* in the monography by Nagata[1].

On the contrary description of the flow stirred systems formed by the solid phase and liquid is from the point of view of hydrodynamics in literature relatively scarce[2–4]. But with the increasing significance of continuous processes the use of these systems is also increasing. There appear also problems with these systems concerning the dependences of concentration and solid particle size distribution on time at the outlet from the mechanically stirred vessel.

Solution of these problems is usually based on the deterministic approach. But it is known that the statistical nature is one of their expressive features. We will take these processes in this way and will describe them accordingly.

This paper is a contribution to the study on time dependence of concentration of solid particles at the outlet from the flow stirred system from the point of view of application of random processes. Such studies of the two-phase flow systems can serve, together with the kinetic parameters, to the design of units such as chemical reactors, crystallisers *etc.*

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THEORETICAL

The object of this study is the Newtonian liquid charge, containing the system of solid particles. The charge is situated in the vessel of cylindrical shape equipped with the rotary mixer situated in the axis of symmetry of the vessel. The particles are small in comparison with the dimensions of the vessel in which the charge is situated. The mixer rotates so intensively that the hydrodynamic regime is turbulent. It is thus possible to consider that both the liquid motion and motion of carried particles is random. The liquid together with solid particles is withdrawn from a certain location of the change at stationary hydrodynamic regime. The volume of the suspension is kept constant by addition of liquid into the rotor region of the mixer. Motion of particles in the vessel space is in general anisotropic due especially to gravitational force acting on them and resulting in nonuniform distribution of concentration.

It is obvious that the described situation is rather complicated and that it is necessary to introduce simplifying assumptions for its mathematic description. The procedure begins with the investigation of motion of one particle in the liquid at turbulent regime and applying this motion to the whole system of particles. The momentum balance equation for motion of a small spherical particle in the liquid has been derived by Tchen[5] at some simplifying assumptions. Such equation is complicated, its solution complex and frequently impossible.

As has been already stated, it is possible to consider the motion of the liquid and particles as random. The motion of a solid particle in the mechanically stirred charge is thus described from the aspect of random processes so that the forces in the Tchen[5] relation, characterizing interactions between particles and liquid (except of viscous friction forces) have been substituted by only random force. Relations which have been so obtained describing the motion of a single particle were then generalized by the law of large numbers to the suspension of solid particles in liquid. Another simplification is the assumption that concentration of particles is approximately equal in the horizontal cross section of the stirred charge *i.e.* that it is changing significantly only in the vertical direction *i.e.* in the direction of gravitational force.

Description of Motion of Solid particle in Turbulent Liquid

The cylindrical coordinate system is used. With regard to the last assumption we will consider only the change in concentration of suspension in the vertical direction *i.e.* the onedimensional problem (Fig. 1).

For derivation of the stochastic differential equation expressing the motion of particles in turbulent liquid from the point of view of Markov's processes[6], we consider the position $z(t)$ of particle as the random function of time t. For the velocity of mo-

tion of the particle then holds

$$V(t) = \mathrm{d}Z(t)/\mathrm{d}t\,. \qquad (1)$$

For simplification of the description, the motion of the liquid charge which is surrounding the particle is not considered and its interaction with the particle is substituted by the action of external forces. Thus we assume that the following forces act on the particle:

1) Force proportional to the velocity of particle motion with respect to the liquid and counter to the direction of motion of the particle. This force is from physical point of view the force of viscous resistance. *2*) Force, independent both of velocity and position of particle and oriented counter to the positive direction of the axis z. This force represents the gravitational field effect on the particle. *3*) Random force whose impuls is proportional to the Wiener process[6] $W(t)$ and which is characterizing the effect of turbulence on the particle.

On basis of the made assumptions *1*) to *3*) it is possible to write the differential momentum balance in the form

$$\mu\,\mathrm{d}V(t) = -\alpha\,V(t)\,\mathrm{d}t - \beta\,\mathrm{d}t + \gamma\,\mathrm{d}W(t)\,, \qquad (2)$$

where μ is the mass of the considered particle and α, β, and γ are constants at steady operating conditions in the equipment. We substitute into the first right hand side term of Eq. (*1*) and we neglect the random velocity changes with respect to random changes of the particle position. It has been demonstrated[7] that the last assumption

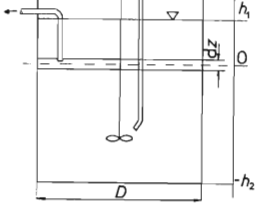

FIG. 1
Mixer with Random Particle Motion

holds the more accurately the longer time passes from the beginning of operation, *i.e.* from the moment the particle has been in a certain position (*e.g.* on the bottom of the vessel.)

By the given simplification of Eq. (*2*) the stochastic differential equation is obtained

$$\mathrm{d}Z(t) + \omega\,\mathrm{d}t - \varepsilon\,\mathrm{d}W(t) = 0\,, \tag{3}$$

where $\varepsilon = \gamma/\alpha$ and $\omega = \beta/\alpha$.

It is possible to prove, that the given stochastic differential equation[3] describes the diffusion Markov's process, *i.e.* such Markov's process where the considered random functions are continuous for every value of the variable t. For this process holds in general the Kolmogorov's forward equation[6] which for the case of Eq. (*3*) can be written in the form

$$\frac{\partial f}{\partial t} - \omega\frac{\partial f}{\partial z} - \frac{\varepsilon^2}{2}\frac{\partial^2 f}{\partial z^2} = 0\,, \tag{4}$$

where $f = f(z;\, t/\xi;\, \tau)$ is the transitive probability density. It is obvious from Eqs (*3*) and (*4*) that the constant ω is the terminal velocity of particle in the quiescent liquid and the quantity $\varepsilon^2/2$ represents the turbulent diffusivity.

Equation (*4*) is the partial differential equation of parabolic type, which can be solved analytically under the given initial and boundary conditions. By solving this differential equation, the value of the probability density of the random function $Z(t)$ is obtained. It has been demonstrated[8], that at certain assumptions and with regard to the law of large numbers[7] this probability density is directly proportional to the concentration of solid particles. The symbol f in Eq. (*4*) can be thus formally considered to be this quantity.*

Determination of Initial and Boundary Conditions

We select a onedimensional coordinate system with the origin in the point of withdrawal of suspension (Fig. 1). The boundaries of the system are then determined by the coordinates h_1 and $-h_2$. Let us moreover assume that the height of the quiescent liquid in the vessel is equal to its diamter D. There obviously holds the relation

$$h_1 + h_2 = D\,. \tag{5}$$

* The term concentration of solid phase in a point is considered in the usual sense: "by point" is meant a prism with the centre in the given point so large as to take the mass of solid phase in the volume of the prism as the continuous function of location and time, with a sufficient accuracy.

The origin of the coordinate system is, in the frame of our onedimensional model, a point from where the solid phase is sucked off. This fact can be in general the cause of singularity of the function $f(.)$ in this point. Thus in our next considerations the vessel is divided into two regions: region situated above the plane passing through the origin and the region situated below it and each will be described separately.

At first the corresponding dimensionless quantities are introduced. The dimensionless time θ

$$\theta = t\omega/D \tag{5a}$$

dimensionless length coordinate

$$y = z/D \tag{5b}$$

dimensionless concentration (as the mass of the solid phase in the point y and in time θ, related to the total initial mass of solid phase in the charge)

$$C(y, \theta) = Df(yD; \theta D/\omega) \tag{5c}$$

and the dimensionless coordinates of boundaries of the system

$$s = h_1/D\,; \quad s - 1 = -h_2/D\,. \tag{5d}$$

By this way the following relations are obtained from Eq. (4)

$$\frac{\partial C_1}{\partial \theta} - \frac{\partial C_1}{\partial y} - K\frac{\partial^2 C_1}{\partial y^2} = 0 \quad [0 \leqq y \leqq s] \tag{6a}$$

and

$$\frac{\partial C_2}{\partial \theta} - \frac{\partial C_2}{\partial y} - K\frac{\partial^2 C_2}{\partial y^2} = 0 \quad [s - 1 \leqq y \leqq 0]\,, \tag{6b}$$

where the constant K is characterized by the ratio of turbulent diffusivity and terminal particle velocity

$$K = \varepsilon^2/(2D\omega)\,. \tag{7}$$

The system from the point of view of mass transfer is on both ends closed. As concerns the random processes the reflexive boundaries[9] are concerned, for which there holds

$$C_1 + K\frac{\partial C_1}{\partial y} = 0\,; \quad [y = s,\ \theta > 0] \tag{8a}$$

and

$$C_2 + K \frac{\partial C_2}{\partial y} = 0 \,. \quad [y = s - 1,\ \theta > 0] \,. \tag{8b}$$

Moreover it is necessary to characterize the bonding conditions in the singular point. First of all there obviously holds equality of concentrations

$$C_1 = C_2 \quad [y = 0;\ \theta > 0] \,. \tag{9a}$$

Beside this there holds the material balance of the solid phase with respect to the volume of infinitesimal thickness dz, denoted by hatching also in Fig. 1 which can be written

$$wS_t c_1 = \left(\omega c_1 + \frac{\varepsilon^2}{2} \frac{\partial c_1}{\partial z} - \omega c_2 - \frac{\varepsilon^2}{2} \frac{\partial c_2}{\partial z} \right) . S_n \,,$$

where S_n is the cross-sectional area of the vessel, S_t cross-sectional area of the pipe and w velocity of liquid in this pipe. After arrangement of this relation we obtain the last bonding condition

$$\frac{M}{K} C_1 = \frac{\partial C_1}{\partial y} - \frac{\partial C_2}{\partial y} \,, \quad [y = 0;\ \theta > 0] \,, \tag{9b}$$

where

$$M = \frac{w}{\omega} . \frac{S_t}{S_n} \,. \tag{10}$$

Initial condition gives distribution of the solid phase in the vessel in time $\theta = 0$. Practically two possibilities come into consideration: *a*) At the beginning of the experiment is the solid phase concentrated at the bottom of the vessel, which can be expressed by the relation

$$C_0 = C_\delta = \delta(y - 1 + s) \quad [\theta = 0] \,. \tag{11}$$

b) Solid phase is at first distributed by mixing so that the stationary state is reached and then from the moment $\theta = 0$ the withdrawal of product is started. The corresponding explicite relation can be obviously obtained as the stationary solution of any of Eqs (*6*), as without withdrawal of suspension the function $C_0 = C_p = C_1 = C_2$ in the point $y = 0$ has no singularity:

$$K \frac{\mathrm{d}^2 C_p}{\mathrm{d}y^2} + \frac{\mathrm{d}C_p}{\mathrm{d}y} = 0 \quad [s - 1 \leqq y \leqq s;\ \theta \leqq 0] \,. \tag{12}$$

One boundary condition is any of relations (*8*), the second one is obtained on basis of the basical consideration that the total mass is in this case constant

$$\int_{s-1}^{s} C_p(y)\,\mathrm{d}y = 1\,. \tag{13}$$

Solution of Eq. (*11*) is then given by the relation

$$C_p(y) = \exp(-y/K)/\{K \exp(-s/K)\,.\,[\exp(1/K) - 1]\} \quad [\theta = 0]\,. \tag{14}$$

So is the problem fully determined, at the given assumptions is the result of the model a onedimensional diffusion equation with the "convective" term (drift term) but with unusual boundary conditions.

Solution of Diffusion Equations

The system of diffusion equations (relations (*6*)) can be solved analytically by the Fourier method[10].
The particular solution is obtained in the form

$$C_j^0(y, \theta) = \exp[-\lambda_j^2\theta - y/(2K)]\,[A_j \cosh(pa_j y) + pB_j \sinh(pa_j y)]\,, \tag{15}$$

where

$$\lambda_j^2 = 1/(4K) - K(pa_j)^2\,. \quad [j = 1, 2]\,. \tag{16}$$

The quantities A_j, B_j and a_j are real numbers, whose values must be determined from initial and boundary conditions. The quantity p can equal only to one or to $i = \sqrt{-1}$ so that the right hand side of Eq. (*16*) would always be non-negative. From the binding condition (*9a*) is obtained directly

$$A_1 = A_2 = A\,; \quad a_1^2 = a_2^2 = a^2\,.$$

From boundary conditions (*8*) it is possible to form relations

$$B_j/A = \varphi_j/p = -\frac{\cosh(pay_j) + 2Kpa \sinh(pay_j)}{p[2Kpa \cosh(pay_j) + \sinh(pay_j)]}\,, \tag{17}$$

where $y_1 = s$ and $y_2 = s - 1$.
From the binding condition (*9b*) the relation is obtained

$$M = (\varphi_1 - \varphi_2)\,.\,Kpa \equiv g(a)\,. \tag{18}$$

The last equation with regard to relation (*17*) is transcendent equation with respect to a.

With regard to familiar relations $\sin x = -i \sinh(ix)$ and $\cos x = \cosh(ix)$, where $i = \sqrt{-1}$ it is possible to demonstrate easily that in the case $p = \sqrt{-1}$ the function becomes also only equal to real values and that it is periodical with the 2π. Beside this is the function $g(a)$ symmetrical with respect to the origin. By solving Eq. (*18*) an infinite number of roots is obtained, two of which differ mutually only by the sign. In Fig. 2 is plotted qualitatively the function $1/g(a)$ for the given value of parameter K for both values of p. In point G the function becomes equal to $g(a)$ as can be easily proved

$$\lim_{a \to 0} g(a) = 1/A = [(2K + s)^2 - (2K + s)]/K . \qquad (19)$$

From this plot is obvious that for smaller values M *i.e.* in the case $1/M > G$ is the first root the solution of Eq. (*18*) for $p = 1$, *i.e.* in the relation appear hyperbolic functions. In the case of large values of M, also p is for the first root equal to the imaginary unit. In the special case is M equal to $1/G$, the first root is twice as large and becomes equal to zero. All other roots are solutions of Eq. (*18*) in which there appear trigonometric functions, as $p = \sqrt{-1}$.

The general solution can be thus written in the form of an infinite sum, while with regard to the statement on symmetry of the function $g(a)$ only non-zero values of roots a_i are considered

$$C_j(y, 0) = \sum_{i=1}^{\infty} A_i \exp\{-[1/(4K) - K(pa_i)^2]\, 0 - y/(2K)\} \, . \, Y_{ij}(y) , \qquad (20)$$

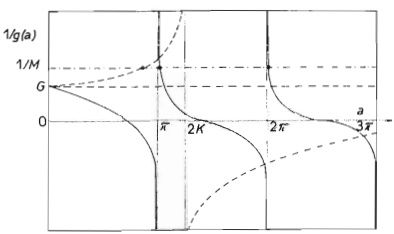

FIG. 2

Plot of Function $1/g(a)$ Defined by Eq. (*18*) ——— $p = \sqrt{-1}$, — — — $p = 1$
In point G there holds $1/g(a) = K/[(2K + s)^2 - (2K + s)]$.

where

$$Y_{ij}(y) = \cosh(pa_i y) + \varphi_j \sinh(pa_i y) \quad [j = 1, 2] .$$

As results from Eqs (*6*) is the index $j = 1$ related to the region above the plane in which the withdrawal of suspension takes place the index $j = 2$ to region under this plane.

The values of constants A_i must be determined from initial condition which is given by the Eq. (*11*) or (*14*). The method of orthogonal functions is used in this case (*e.g.*[10]). But in our case is orthogonal, unlike usual procedures, always a pair of functions describing the situation simultaneously above and below the plane of withdrawal[11]

$$A_i = \frac{\int_0^s C_0(y) \exp[y/(2K)]\, Y_{i1}(y)\, dy + \int_{s-1}^0 C_0(y) \exp[y/(2K)]\, Y_{i2}(y)\, dy}{\int_0^s Y_{i1}^2(y)\, dy + \int_{s-1}^0 Y_{i2}^2(y)\, dy} . \qquad (21)$$

After this given integration the values of constants A_i are obtained. Let us write

$$A_i = F_i/E_i , \qquad (22)$$

where F_i is numerator and E_i denominator of the fraction (*21*). It is obvious that the value E_i is independent of the initial condition. It is possible to demonstrate[11] that there holds

$$E_i = \psi_i(y_1) - \psi_i(y_2) - 2M/(Ka_i p) , \qquad (23)$$

where

$$\psi_i(y_j) = \sinh(2a_i p y_j) . (1 + \varphi_j^2/p^2) + 2(\varphi_j/p) \cosh(2a_i p y_j) + 2a_i p y_j (1 - \varphi_j^2/p^2) ; \quad [j = 1, 2] , \qquad (24)$$

and where φ_j/p and y_j are defined by Eq. (*17*).

The value of numerator F_i depends on initial condition. For conditions defined in Eq. (*11*) or (*14*) the relation is obtained

$$F_i = \begin{cases} 2Kpa_i/\{2Kpa_i \cosh(pa_i y_2) + \sinh(pa_i y_2)\} & (11) \\ 16a_i pM/\{[1 + (2Ka_i p)^2][\exp(1/K] - 1] \exp(-s/K)\} & (14) . \end{cases} \qquad (25)$$

From Eqs (*23*)–(*25*) it is substituted into Eq. (*22*) and from there into Eq. (*20*). In this way the problem is completely solved.

DISCUSSION

The proposal procedure demonstrates application of the stochastic differential equation to description of the random process, *i.e.* of a procedure which is more and more frequently used for description of chemical engineering processes (see *e.g.*[12]). This procedure leads in our case to the classical parabolic onedimensional differential diffusion equation, used in chemical engineering for description of heat and mass transfer in a single phase system.

On the proposed model according to our opinion are especially interesting the unusual boundary conditions expressing the internal withdrawal of material from the system. In general it would thus be possible to use this procedure for the onedimensional description of flow reactors with internal product withdrawal. In case the internal product withdrawal is altered to the end withdrawal, the boundary conditions are reduced to the outlet Danckwerts condition[8] (at the assumption that the rate of withdrawal equals to the terminal particle velocity).

Physical Sense of the Model

The model describes distribution of particle concentration along the vessel height with time. The model is onedimensional and is thus not describing the radial distribution of concentration which is *e.g.* due to liquid convection (circulation). In other words the constant concentration of solid phase in the given height is assumed.

In our concrete case the coordinate system is selected so that its origin is situated in the point of product withdrawal. This selection enables simplification of resulting relations for description of time changes of concentration at the outlet from the vessel.

The quantity $\varepsilon^2/2$ from Eq. (*4*) is usually denoted, as has been already given above, as turbulent diffusivity. It is in this case usual to assume that in the charge stirred by the mechanical rotary mixer this quantity is proportional to the product of stirring rate and second power of its diameter. The coefficient ε^2 is a part of the dimensionless parameter K (Eq. (*7*)), which is characterizing the ratio of velocity by which are the particles suspended to the sedimentation rate or ratio of forces causing turbulence of liquid to the gravitational force acting on the particle.

The parameter M, defined by Eq. (*10*) is characterizing the ratio of rate of suspension withdrawal to the terminal velocity of particles.

The parameter s is the dimensionless distance of the point of withdrawal from the liquid surface in the vessel.

Behaviour of the Model at Limiting Values of Parameters

How correct is the model – in the range of given assumptions – it is usual to consider according to relations obtained at limiting values of parameters, *i.e.* in situations

usually clear from the physical point of view. Let us discuss several such cases here.

a) $K \to \infty$. For particles of final dimensions the turbulence in the charge must obviously increase infinitely. It is thus possible to expect that the device will behave as an ideal mixer.

Let us prove this statement. From discussion of Eq. (*19*) or from Fig. 2 it is obvious that in this case in relation (*18*) there appear only trigonometric functions. Thus we put $p = \sqrt{-1}$ and from Eqs (*17*) and (*18*) we obtain after arrangement the equation

$$M = (4K^2a_i^2 + 1)\, Ka_i \operatorname{tg} a_i / \{[1 + \operatorname{tg}(a_i y_1) \,.\, \operatorname{tg}(a_i y_2)] \,.$$

$$. \{4K^2a_i^2 + 2Ka_i[\operatorname{tg}(a_i y_1) + \operatorname{tg}(a_i y_2)] + \operatorname{tg}(a_i y_1) \operatorname{tg}(a_i y_2)\}\} = g(a_i) \,. \quad (26)$$

Also we investigate the limit of the right hand side of Eq. (*26*) for $K \to \infty$ and for the limited positive M. The limit is satisfied for values of roots

$$a_i = (i - 1)\,\pi \,. \quad (27)$$

The first root is obviously equal to zero. Its convergence at the increase of K beyond all limits is studied later. Obviously there holds

$$M = \lim_{K \to \infty} g(a_1) = \lim_{K \to \infty} Ka_1 \operatorname{tg} a_1 = \lim_{K \to \infty} Ka_1^2$$

and thus

$$a_1^2 = \lim_{K \to \infty} M/K \,. \quad (28)$$

By substituting from these equations into corresponding relations the values of relations are obtained

$$A_1 = 1 \,; \quad A_i = 0 \quad [i = 2, 3, \ldots] \,; \quad Y_{1j} = 1 \,,$$

so that Eq. (*20*) takes the form

$$\lim_{K \to \infty} C_j(y, \theta) = \lim_{K \to \infty} \exp\{-[1/(4K) + K(M/K)]\,\theta - y/(2K)\} = \exp(-M \,.\, \theta) \,. \quad (29$$

Thus the expression converges to the exponential function with respect to the dimensionless time θ and is not a function of spacial coordinate. Thus the ideal mixer is actually concerned.

b) $K \to 0$. The turbulence in the charge and thus the force which lifts the particles is equal to zero. Thus the particles will always lay on the bottom, *i.e.* for all values of θ the function $C_j(y, \theta)$ is equal to the initial condition $C_\delta(y)$. This fact can be proved by direct solution of differential Eqs (*6*). Also the convergence of the stationary initial condition proves this statement

$$\lim_{K \to 0} C_p(y) = C_\delta(y) .$$

c) $M \to 0$. This case demonstrates the fact that the solid phase is not withdrawn from the charge. It is obviously possible to reach the stationary distribution of the solid phase in the vessel in this case. As results from condition (*9b*), the function C_j always has continuous derivation and thus $C_1 = C_2$. It is thus possible to solve only one of Eqs (*6*) at the boundary conditions (*8*) whose stationary solution is relation (*14*). By similar procedure as in case sub *a*), it is possible to prove hereby that there holds

$$\lim_{M \to 0} A_1 = \left[K \exp\left(-s/K\right) \exp\left(1/K\right) - 1\right]^{-1} ; \quad \lim_{M \to 0} A_i = 0 ; \quad [i = 2, 3, \ldots] . \quad (30)$$

The corresponding roots become equal to values

$$\lim_{M \to 0} a_1 = 1/(2K) ; \quad \lim_{M \to 0} a_i = (i - 1)\,\pi ; \quad [i = 2, 3, \ldots] . \quad (31)$$

The proposed function thus corresponds in limiting cases to the physical model.

Material Balance of the System

From the criticism of the model, given at the beginning of this chapter it is obvious that the proposed relations will not be able to describe the actual distribution of particles in the space of the stirred charge with the sufficient accuracy. But it is possible to assume that the integral characteristics *i.e.* the total withdrawal of the solid phase as the function of time will satisfactorily agree with the actual situaton. The model thus enables description of the wash-out of solid phase from the system.

Let us put together the material balance of the considered system in dimensionless form expressing the fact that the decrease in the solid phase content is just equal to the withdrawn amount

$$-\frac{\mathrm{d}}{\mathrm{d}\theta} \int_0^s C_1(y, \theta)\,\mathrm{d}y + \int_{s-1}^0 C_2(y, \theta)\,\mathrm{d}y = -\frac{\mathrm{d}}{\mathrm{d}\theta}\, m(\theta) = MC(0, \theta) . \quad (32)$$

Is is obvious from Eq. (*20*) that the solid phase concentration in the point of its withdrawal (*i.e.* in the point $y = 0$) is equal to

$$C(0, \theta) = \sum_{i=1}^{\infty} A_i \exp\{-[1/(4K) - K(pa_i)^2]\theta\} = \sum_{i=1}^{\infty} A_i \exp(-\lambda_i^2\theta) . \qquad (33)$$

The dimensionless mass $m(\theta)$ defined as the actual mass in the moment θ, related to the initial mass of the solid phase in the charge is obtained by integration of Eq.(*32*) in the form

$$m(\theta) = M \sum_{i=1}^{\infty} A_i \exp(-\lambda_i^2\theta)/\lambda_i^2 . \qquad (34)$$

From this last relation results that the infinite sum is always limited, $m(0) = M \sum_{i=1}^{\infty} A_i/\lambda_i^2 = 1$.

The relation (*33*) enables description of wash-out of the solid phase. For conditions at which the system is approaching the ideal state this expression can be simplified. We introduce the term separation factor ϱ as the ratio of solid phase concentrations in the point of withdrawal and in the time θ to the mean concentration of the solid phase in the charge

$$\varrho(\theta) = \frac{(s - s + 1)\, C(0, \theta)}{\int_0^s C_1 \, dy + \int_{s-1}^0 C_2 \, dy} = \frac{C(0, \theta)}{m(\theta)} . \qquad (35)$$

It is obvious that the so defined factor is a function of time. For sufficiently large K or for small values of M – as we have demonstrated in the last paragraph – the constants A_i with the exception of the first one are approaching zero, so that after substitution from Eqs (*33*) and (*34*) into (*35*) the relation is obtained $\varrho \approx \lambda_1^2/M =$ = konst.

and from here the relation results

$$C(0, \theta) = (\lambda_1^2/M) \exp(-\lambda_1^2\theta) . \qquad (36)$$

The relation (*36*) takes the form of Eq. (*29*) with further rise of K above all limits. The relation (*36*) is also exponential with respect to the dimensionless time θ, but the value of the separation factor depends on the flow rate of liquid through the system and on the turbulence intensity and terminal particle velocity.

We suggest to call such regime quazi-ideal. Bourne and Sharma[3] as well as Baldi and Conti[4] have obviously been experimenting under conditions close to this regime. In both cases the experimental wash-out curves demonstrate the exponential depen-

dence on time but they do not correspond to the ideal mixer with the mean residence time of liquid in the equipment equal to the ratio of its flow rate to the volume of the charge.

CONCLUSIONS

From the analysis of the proposed model results that it can describe some effects in the hydrodynamics of stirred systems solid phase–liquid, first of all the time changes of mass of the solid phase in the charge. It is necessary to mention that in the given form the model holds accurately only for monodisperse solid particles. Its extension to polydisperse systems of solid particles is one of its prospective improvements.

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LIST OF SYMBOLS

A	constant in Eq. (*15*)
a	root of transcendent Eq. (*18*)
B	constant in Eq. (*15*)
$C = c\,S_n(h_1 - h_2)/\mu(0)$	dimensionless concentration
c	concentration of solid phase, kg m^{-3}
D	diameter of vessel, m
E	expression defined by relation (*23*)
F	expression defined by relation (*25*)
f	transitive probability density, m^{-1}
g	function defined by relation (*18*)
h_1	upper end of the stirrer, m
h_2	lower end of the stirrer, m
K	constant defined by Eq. (*7*)
M	constant defined by Eq. (*10*)
$m = \mu(t)/\mu(0)$	dimensionless mass of solid phase
p	real or imaginary unit
$s = h_1/D$	dimensionless boundary of the mixer
S_n	cross-sectional area, m^2
t	time, s
V	velocity of particle (random function of time), m s^{-1}
W	Wiener process, s$^{1/2}$
w	velocity of liquid in the outlet tube, m s^{-1}
Y	function defined by Eq. (*20*)
$y = z/D$	dimensionless length coordinate
Z	position of particle (random function of time), m
z	length coordinate, m
α	constant characterizing friction force, kg s^{-1}
β	constant characterizing the gravitational force, kg m s^{-2}
γ	constant characterizing random force, kg m s$^{-3/2}$
δ	Diracs function

ε	constant characterizing turbulent diffusivity, $m\,s^{-1/2}$
$\theta = t\omega/D$	dimensionless time
λ	constant defined by Eq. (*16*)
μ	mass of particle, kg
$\mu(0)$	initial mass of solid phase in the charge, kg
$\mu(t)$	instantaneous mass of the solid phase in the charge, kg
ξ	initial position of the particle, m
ϱ	separation factor
τ	initial moment, s
φ	function defined by Eq. (*17*)
ψ	function defined by Eq. (*24*)
ω	terminal velocity of particle, $m\,s^{-1}$

Subscripts

1 related to the volume of the vessel above the plane of sample withdrawal
2 related to the volume of the vessel below the plane of sample widthdrawal
i summation index (natural number)
n related to the vessel
o related to initial conditions
p related to stationary state
δ related to bottom of the vessel

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